

## Characteristic Analysis of Insurance Stochastic interest rate based on nonparametric estimation Statistical Model

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**Abstract:** How to adapt to the participation of migrant workers in China's basic old-age insurance system is an important challenge brought by large-scale population migration, of which the most important reform is to realize the transformation of social endowment insurance relationship. the biggest difficulty in the transition of old-age insurance relationship is the transfer of funds. In this paper, the stochastic interest rate which obeys the Hull-White model is introduced, and the insurance actuarial pricing of European options in the generalized B-S model is discussed. By using the actual probability measure of the underlying asset price process and the fair premium principle, it is obtained that there are two cases in which there is no dividend payment during the option validity period. The insurance actuarial pricing formula of European options. considering that the insurance pricing of options depends on unknown model parameters-the volatility of the underlying asset price, the drift parameters of the stochastic interest rate process and the volatility parameters, using the observed data of asset prices and random interest rates, the insurance actuarial pricing formula based on model parameter estimation is given, and the consistency of the pricing formula is discussed. based on a standard person hypothesis. This paper empirically simulates the fund transfer dilemma of the current social endowment insurance relationship transition policy. The policy of non-transfer of funds reduces the potential benefits of the basic pension of the insured individuals, while the policy of capital transfer has the technical problem of discounting the risk of death and the realistic dilemma of uneven bitterness and happiness in different places. The fundamental way to solve the predicament and realize the effective transformation of social endowment insurance relationship is to realize the overall planning of the whole country.

### 1. Introduction

The actuarial method of option pricing was first proposed by Mogens Bladt and Tina Hviid Rydberg [1] in 1998. Since the actuarial method does not have any market hypothesis, it is not only applicable to the balanced, complete and arbitrage-free financial market, but also effective for the unbalanced, incomplete and arbitrage-free financial market. Literature [2] studies the option pricing based on actuarial method in the generalized B-S model. Other researchers have also conducted in-depth studies on the actuarial method of option insurance [3-4]. The risk-free interest rate in the above literatures is a definite function of time, but a large number of empirical analysis shows that interest rate has the feature of mean recovery in modern financial market. Therefore, taking interest rate only as a definite function of time cannot describe the actual change characteristics of interest rate well. Literature [5] gives an actuarial pricing method for European option and exchange option under stochastic interest rate and Ornstein-Uhlenback model.

In view of this, this paper studies the actuarial pricing of European options in the generalized B-S model with random interest rate. Firstly, the risk-free interest rate that follows the Hull-White model is introduced, and the actuarial pricing formula for European options with or without dividend payment is obtained by using the actual probability measure of the underlying asset price process and the principle of fair premium. Then, considering that the insurance pricing of options

depends on unknown model parameters, on the one hand, the strong consistency estimator of the price volatility of risky assets is constructed by using the observed data of risky asset prices. On the other hand, under the condition that the risk-free interest rate model satisfies the local stationary process, the consistent estimators of drift parameters and volatility parameters in the process of random interest rate are obtained by using weighted least squares method and Kolmogorov forward equation based on the observation samples of random interest rate. Finally, the actuarial pricing formula for European options is given based on the estimator of the time-varying diffusion model parameters, and the consistency of the pricing formula is discussed. The actuarial pricing formula of option insurance obtained in this paper can be directly applied to financial practice, which improves the effectiveness and convenience of option pricing formula in practical application.

## 2. Market models and fundamentals

Consider that there are two kinds of assets in the financial market, one is risky assets (such as stocks), and the other is risk-free assets (such as bonds). It is assumed that the price of risky assets  $\{S_t, t \geq 0\}$  is a random process defined on a complete filter space, which satisfies the following variable coefficient Black Scholes model.

$$\frac{dS_t}{S_t} = \mu(t)dt + \sigma(t)dB_t \quad (1)$$

The stochastic differential equation that the price process of risk-free assets  $\{P_t, T \geq 0\}$  satisfies is  $dP_t = r(t)P_t dt$ , where  $R(t)$  is the risk-free interest rate at time  $T$ , which satisfies the Short-period interest rate model of Hull White.

$$dr(t) = (\alpha(t) + \beta(t)r(t))dt + \sigma_r(t)dW_t \quad (2)$$

$\alpha(t), \beta(t), \sigma_r(t)$  Is a function of time  $t$ , and the parameter  $(t)$  describes the long-term average of interest rates, The expected rate of return of risk asset price process  $\{S_t, T \geq 0\}$  over time interval  $[0, t]$

$$\exp\left(\int_0^T \psi(t) dt\right) = \frac{E(S_T)}{S_0} \quad (3)$$

The asset of European option actuarial value is defined as: option is performed, maturity of the underlying asset price discounted and the difference between the strike price of discounted in the underlying asset price under the real probability measure of mathematical expectation, including risk assets (such as the price of the underlying asset) according to its expected return (e.g., (3) defined) discount, risk-free asset prices (such as the strike price) by the risk-free interest rate discount.

## 3. Actuarial pricing of options at random interest rates

Actuarial pricing of European options in the Hull-White stochastic interest rate model with the generalized Black-Scholes model. Firstly, the following lemma is given.

Set the random variable  $\xi \sim N(0,1), \eta \sim N(0,1)$  and  $\text{Cov}(\xi, \eta) = \rho$

$$E[\exp(c\xi + d\eta)I_{\{a\xi + b\eta \geq k\}}] = \exp\left(\frac{1}{2}(c^2 + d^2 + 2\rho cd)\right)\Phi\left(\frac{ac + bd + \rho(ad + bc)}{\sqrt{a^2 + b^2 + 2ab\rho}}\right) \quad (4)$$

Assuming that the price process  $\{S_t, T \geq 0\}$  of risky assets satisfies the model (1), the risk-free interest rate process  $\{r(t), t \geq 0\}$  satisfies the short-term interest rate model (2), and the risky assets have no dividend payment during the term of option, the actuarial pricing formulas of European call option and European put option are as follows:

$$C(K, T) = S_0\Phi(d_1) - K\exp\left(\frac{1}{2}\int_0^T \sigma_r^2(t)h^2(t, T)dt - H(T)\right)\Phi(d_2) \quad (5)$$

## 4. Research hypothesis

### 4.1 Initial Model

In this paper, it is assumed that A male insured H continuously participates in the employment insurance in A, B and C successively. The insured age is A, B and C respectively, and  $A + B + C = 35$ . When he reaches the age of 60, he will retire and start to receive pension. In order to eliminate the uncertainty of wage change, standardized the insured's wage premium is the average of A, B and C positions Wages, and assume that the three salaries are the same and the growth rate is the same. This hypothesis means that if H's basic pension benefits in A, B and C all meet the acquired benefit rule in Equation (3) when  $15 \leq H \leq 35$ , then theoretically the sum of H's benefits in A, B and C is the same as the basic pension benefits obtained by H after 35 years of continuous participation in any one place. Then, according to the basic pension benefit calculation formula, the base value of the total basic pension benefit of H insured in any place A, B and C for 35 years can be calculated as follows:

$$P = 0.35 \cdot W_{59} \sum_{n=e}^{\omega} (1 + \delta)^{n-60} \sigma^{n-60} \prod_{m=60}^n p_m \quad (6)$$

### 4.2 Initial Model

Case 1: H has been insured in Place A for less than 15 years and has been discontinued and fails to renew the insurance thereafter.

According to formula (3) of the earned beneficial right rule of job insurance, H will not be able to get the basic pension benefits, and the social pooling account contribution income R obtained by the pooling fund in A is:

$$R^A = 0.16 \sum_{n=25}^{25+a} w_n^A \quad a < 15 \quad (7)$$

Situation 2: H has been insured in A and B for less than 15 years in total and has lost the insurance and failed to renew the insurance thereafter.

Similarly, H will not be eligible for basic pension benefits under the priority-of-entitlement rule. In addition, according to the fund transfer policy, the social pooling account contribution income R obtained by the pooling fund of A and B is:

$$\begin{cases} R^A = 0.04 \sum_{n=25}^{25+a} w_n^A \\ R^B = 0.12 \sum_{n=25}^{25+a} w_n^A + 0.16 \sum_{n=26+a}^{26+a+b} w_n^B \end{cases} \quad a + b < 15 \quad (8)$$

### 4.3 Discussion and analysis

Cognitive ability was severely damaged also widely to define the state of the long-term care is an important indicator of the CLHLS database on cognitive investigation including two problems: one is based on simple cognitive function scale (MMSE) set about reaction ability, attention and comprehension, memory, and language and self coordination problems; The second is about whether you have the following mental and memory-related diseases (Parkinson's, dementia, epilepsy, etc.). Because the MMSE test is prone to false responses, this paper adopts the definition of cognitive dysfunction proposed by Wang Xinjun and Wang Jiayu (2018). If the elderly sample has emotional, mental and memory-related diseases identified by the hospital, it will be considered as having cognitive dysfunction.

This paper proves that there are practical problems whether the funds are transferred or not between different pooling areas. If funds are not transferred, the basic pension rights of insured individuals will be damaged. If the transfer of funds, not only is it difficult to solve the technical problem of death risk discount, but also each region will face the realistic dilemma of imbalance of fund income and expenditure. Therefore, the current practice of transferring part of the social pooling account funds by 12% of the contribution base is only an expedient measure to balance the interests between the place of transfer and the place of transfer in the short term, which is not

conducive to the sustainable development of the social endowment insurance system in the long run. Return to endowment insurance concern to turn from fund transfer problem to continue itself, the reason that appears equity continues problem, because ginseng is in, plan as a whole in different area mobile obtain employment, produce fund to transfer a problem then. Tracing back to the source, the choice dilemma of capital transfer is caused by the low pooling level. In addition to the problems proved in this paper, a low level of pooling is not conducive to risk sharing and the effect of insurance law of large numbers to reduce the ability of the system to resist risks, while solidifying regional segmentation and raising the operating cost of the system. Therefore, the solution to solve the dilemma of social endowment insurance relationship to continue the transfer of funds is to improve the level of pooling, the fundamental strategy is to achieve national pooling.

## 5. Conclusion

In this paper, the actuarial pricing of European options in the generalized B-S model at random interest rates is studied. Firstly, the actuarial pricing formula of European option with or without dividend payment is discussed by using the actual probability measure of the underlying asset price process and the principle of fair premium. Then, considering the insurance option pricing problem is dependent on the unknown model parameters - the price of the asset volatility, drift in the process of stochastic interest rate and volatility parameters.

The report of the 19th National Congress calls for national pooling of endowment insurance as soon as possible. based on asset prices and the observation data of stochastic interest rate, the estimate of model parameters is given, and the options on the actuarial pricing formula based on the estimator obtained, at the same time, the pricing formula of consistency are discussed. Obviously, the current social endowment insurance relationship to continue the policy is only a transitional approach. In view of the current social endowment insurance relation because of the transfer policy in the process of transferring the potential impact caused by the different parts of the fund balance of payments, in the short term the solution is in increasing employment insurance fund adjustment funds of the central dispensing ratio in the process of transferring to endowment insurance relation in society as a whole account funds transfer ratio as a benchmark, use of relief fund for more due to participants in places where fund balance pressure increasing funds to adjust. At the same time, promote provincial pooling as soon as possible, and for the realization of national pooling experience, establish standards, clear procedures and lay a foundation. The ultimate goal is to achieve national pooling. In this process, it is necessary to further improve the supporting policies of national pooling, including the recognition of payment wage standard, adjustment of the plan and payment method of basic pension benefits, and upgrading of the power of investment management of individual pension accounts

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